The study of double vector charmonium meson production at B-factories within light cone formalism.

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In this paper the processes $e^+e^- \to J/\Psi J/\Psi,\ J/\Psi\psi',\ \psi'\psi'$ are considered in the framework of light cone formalism. An important distinction of this approach in comparison to the approaches used in other papers is that relativistic and leading logarithmic radiative corrections to the cross section can be resummed within light cone formalism. In this paper the effect of this resummation is studied. It is shown that this effect is important especially for the production of higher charmonium mesons. The predicted cross sections are in agreement with the upper bounds set by Belle collaboration.

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I. INTRODUCTION

The measurement of the cross section of the process $e^+e^- \to J/\Psi\eta_c$ at Belle collaboration [1] revealed large discrepancy between the experiment and the leading order NRQCD prediction. Latter measurements of the processes of double charmonium production at B-factories [2, 3] shown that there is disagreement between theory [4, 5, 6] and experiment in other processes. Only in few years it was realized that the contradiction between NRQCD prediction and experimental result for the process $e^+e^- \to J/\Psi\eta_c$ can be resolved if one takes into account radiative corrections [7] and relativistic corrections simultaneously [8, 9].

In addition to NRQCD [10], hard exclusive processes can studied within light cone formalism (LCF) [11, 12]. Within LCF the cross section is built as an expansion over inverse powers of characteristic energy of the process. There are two very important advantages of LCF in comparison to NRQCD. The first one is connected with the following fact: LCF can be applied to study production of any meson. For instance, it is possible to study production light mesons, such as π mesons, or production heavy mesons, such as charmonium mesons. From NRQCD perspective, this implies that LCF resums infinite series of the relativistic corrections to amplitude, which can be very important. The second advantage is that LCF resums very important part of QCD radiative corrections – the leading logarithmic radiative corrections to amplitude $\sim \alpha_s \log(Q)$. This is very important advantage since at high energies the leading logarithmic corrections can be even more important than the relativistic ones.

The first attempts to study double charmonium production at B-factories in the framework of LCF were done in papers [13, 14, 15]. The main problem of these papers is connected with rather poor knowledge of charmonium distribution amplitudes (DA). It should be noted that within LCF the calculation of hard exclusive charmonium production cannot be considered reliable if one has poor knowledge of DAs. Fortunately, lately charmonium DAs became the object of intensive study [16, 17, 18, 19, 20, 21, 22]. In papers [18, 19, 20] the models of DAs for $1S$ and $2S$ states charmonium mesons were proposed. If one uses these DAs to calculate the cross sections of the processes $e^+e^- \to J/\Psi\eta_c, J/\Psi\eta_c', \psi'\eta_c, \psi'\eta_c'$, the agreement with the experiments can be achieved [23]. In present paper these models of DAs will be used.

A lesson that can be learnt from the study of the process $e^+e^- \to J/\Psi\eta_c$ within NRQCD and LCF is that the leading order NRQCD predictions for hard exclusive processes cannot be considered as reliable before the relativistic and radiative corrections are not taken into the account. This paper is devoted to the study of the hard exclusive processes $e^+e^- \to J/\Psi J/\Psi,\ J/\Psi\psi',\ \psi'\psi'$ in the framework of LCF. An important distinction of this paper in comparison to the papers where these processes were studied earlier [24, 25, 26, 27, 28, 29] is that within LCF the relativistic and leading logarithmic radiative corrections to the cross section can be resummed. Thus one can hope that the predictions obtained in this way are more reliable.

This paper is organized as follows. Next section is devoted to the calculation of the cross sections of the processes under study at the leading order approximation of LCF. In the third section $1/s$ corrections to the leading order result will be considered. In the last section the result of the calculation will be presented and discussed.

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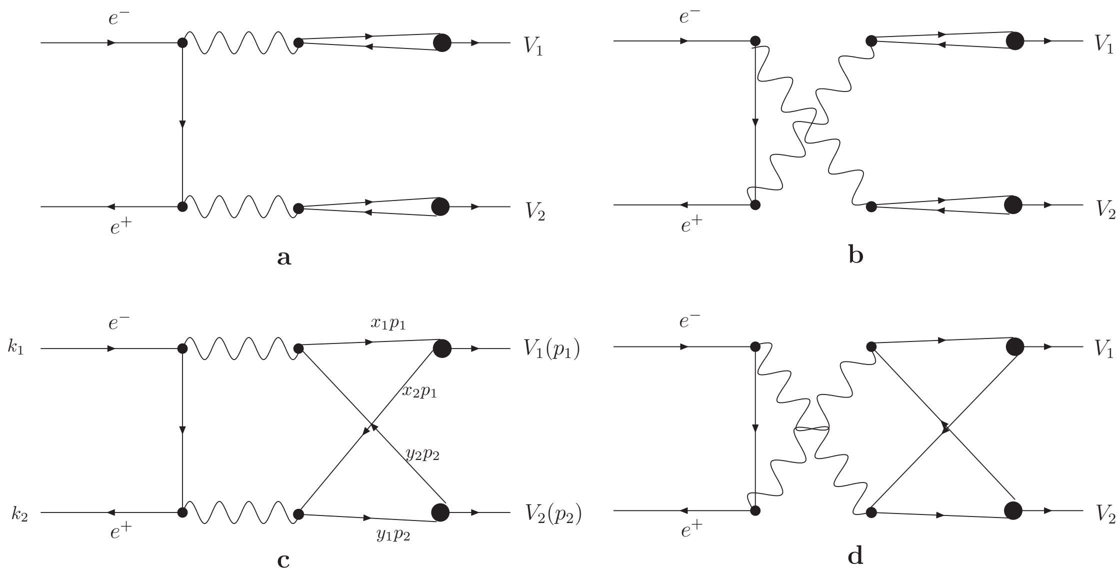

FIG. 1: The diagrams that contribute to the process $e^+e^- \to V_1(p_1)V_2(p_2)$ at the leading order approximation in strong coupling constant.

II. THE LEADING ORDER CONTRIBUTION.

The diagrams that contribute to the process $e^+e^- \to V_1(p_1)V_2(p_2)$ at the leading order approximation in α_s are shown in Fig. 1. The diagrams shown in Fig. 1a, b can be divided into two parts. The first part is the annihilation $e^+e^- \to \gamma\gamma$ which is followed by the fragmentation of photons into vector mesons $V_1(p_1), V_2(p_2)$. Below these diagrams will be referred to as fragmentation diagrams. The second part are the diagrams shown in Fig. 1c, d will be referred to as non-fragmentation diagrams. The cross section $\sigma(s)$ of the process $e^+e^- \to V_1(p_1)V_2(p_2)$ can be written as the sum

$$\sigma(s) = \sigma_{fr}(s) + \sigma_{int}(s) + \sigma_{nfr}(s), \tag{1}$$

where $\sigma_{fr}(s)$ and $\sigma_{nfr}(s)$ are the contributions due to the fragmentation and non-fragmentation diagrams correspondingly, $\sigma_{int}(s)$ is the contribution of the interference between the fragmentation and non-fragmentation diagrams.

In this paper double vector charmonium meson production ($V_i = J/\Psi, \psi'$) at B-factories will be considered. Commonly, to study charmonium production one uses NRQCD formalism [10]. In the framework of NRQCD charmonium mesons are considered as nonrelativistic systems with characteristic velocity $v^2 \sim 0.3$ and the amplitude of charmonium production is the series in the small parameter v^2. The study of double vector charmonium meson production in e^+e^- annihilation within NRQCD was carried out in papers [24, 25, 26, 27, 28, 29].

In this paper light cone formalism [12] will be applied to the study of double vector charmonium meson production. Within this formalism the cross section is the series over inverse powers of characteristic energy of the process. In particular, at the energy $\sqrt{s} = 10.6$ GeV the expansion parameter for the process under consideration is $4M_V^2/s \sim 0.4$.

To begin with let us determine the asymptotic behaviors of $\sigma_{fr}(s), \sigma_{int}(s), \sigma_{nfr}(s)$ in the limit $s \to \infty$. This can be done using the results of paper [28]. In Table I the asymptotic behavior of $\sigma_{fr}(s), \sigma_{int}(s), \sigma_{nfr}(s)$ in the limit $s \to \infty$ for different polarizations of vector mesons (λ_1, λ_2) are shown. From this table one sees that at the leading order approximation in $1/s$ expansion only the fragmentation diagrams with the polarizations $\lambda_1 = \pm 1, \lambda_2 = \mp 1$ contribute. It causes no difficulties to find the expression for this contribution

$$\frac{d\sigma^{fr}_{\pm 1, \mp 1}}{dx} = \frac{16\pi^3\alpha^4 q_c^4 f_1^2 f_2^2 \sqrt{\lambda}}{sM_1^4M_2^4}\left(\frac{1-r_1-r_2}{(1-x^2)\lambda + 4r_1r_2}\right)^2 (1-x^4), \tag{2}$$

where M_1, M_2 are the masses of vector mesons, q_c is the charge of c quark, $x = \cos\theta$, θ is the angle between the momentums of electron and charmonium meson V_1,

$$r_1 = \frac{M_1^2}{s}, \quad r_2 = \frac{M_2^2}{s}, \quad \lambda = 1 + r_1^2 + r_2^2 - 2r_1 - 2r_2 - 2r_1r_2. \tag{3}$$

$V_1(\lambda_1,p_1)V_2(\lambda_2,p_2)$	$\sigma_{fr}(s)$	$\sigma_{int}(s)$	$\sigma_{nfr}(s)$
$\lambda_1=\pm1 \;\; \lambda_2=\mp1$	$\sim 1/s$	$\sim 1/s^2$	$\sim 1/s^3$
$\lambda_1=\pm1 \;\; \lambda_2=0$ $\lambda_1=0 \;\; \lambda_2=\pm1$	$\sim 1/s^2$	$\sim 1/s^3$	$\sim 1/s^4$
$\lambda_1=\pm1 \;\; \lambda_2=\pm1$	$\sim 1/s^3$	$\sim 1/s^4$	$\sim 1/s^5$
$\lambda_1=0 \;\; \lambda_2=0$	$\sim 1/s^3$	$\sim 1/s^3$	$\sim 1/s^3$

TABLE I: The leading behavior of $\sigma_{fr}(s), \sigma_{int}(s), \sigma_{nfr}(s)$ in the limit $s\to\infty$ for the polarization of vector mesons λ_1, λ_2.

The constants f_1 and f_2 are defined through the matrix element of electromagnetic current J_μ^{em}

$$\langle V_i(p_i,\lambda_i)|J_\mu^{em}|0\rangle = q_c f_i \epsilon_\mu^*(\lambda_i). \tag{4}$$

This constants can be determined from the electronic width of vector meson V_i

$$\Gamma(V_i\to e^+e^-) = \frac{4\pi q_c^2\alpha^2 f_i^2}{3M_i^3}. \tag{5}$$

Formula (2) is valid for the production of different mesons. If two identical mesons are produced, this formula must be divided by 2. It should be noted here that formula (2) is in agreement with the result derived in paper [28]. To get the cross section of the process under consideration one should sum over all possible polarizations that give contribution to the cross section at the leading order approximation. Thus up to the corrections $O(1/s^2)$ the cross section is

$$\frac{d\sigma}{dx} = 2\frac{d\sigma^{fr}_{\pm1,\mp1}}{dx} + O\left(\frac{1}{s^2}\right) \tag{6}$$

In this section the cross section of the process $e^+e^-\to V_1(p_1)V_2(p_2)$ has been considered at the leading order approximation in $1/s$ expansion. Strictly speaking, to get the cross section at the leading order approximation one must expand formula (2) in $1/s$ and keep only the first term. However, it turns out that the contribution of the fragmentation diagrams to the amplitude and the cross section can be calculated exactly. So, to improve the accuracy of the calculation done in here, the exact expression for cross section due to the fragmentation diagrams (2) will be used.

In the numerical calculation the following values of the parameters will be used: $M_{J/\Psi}=3.097$ GeV$,\Gamma(J/\Psi\to e^+e^-)=5.55\pm0.14$ KeV$, M_{\psi'}=3.686$ GeV$,\Gamma(J/\Psi\to e^+e^-)=2.48\ \pm0.06$ KeV [30]. The results of the calculation are shown in Table II. In the second colomn one can see the cross sections at the leading order approximation of $1/s$ expansion. In the third colomn the differential cross sections at the leading order approximation are integrated over the region $|\cos\theta|<0.8$.

There are different sources of uncertainty to the values of the cross section at the leading order approximation of $1/s$ expansion. The first one is QCD radiative corrections. These corrections can be divided into three groups. The first group is the radiative corrections due to the exchange of gluons between the quark and antiquark of one charmonium meson. Evidently, the same corrections appear as the radiative corrections to the electronic width of charmonium meson. These corrections are included into the values of the constants f_i, which can determined from the electronic decay width of charmonia (5). So, the uncertainty due to the first group of the radiative corrections is reduced to the experimental uncertainty in the electronic decay widths of charmonia, which is few percents for J/Ψ and ψ' mesons. The next group of the radiative corrections are the corrections due to the exchange of hard gluons between quarks or antiquarks of different mesons. This type of the corrections can be estimated as $\alpha_s^2(E)m_c^2/E^2\sim0.4\%$ [28], where $E=\sqrt{s}/2$, m_c is the mass of c quark. The last group of the radiative corrections is the corrections due to the exchange of soft gluons between quarks or antiquarks of different mesons, which can be estimated as $(m_cv)^4/E^4\sim0.04\%$ [28]. It is seen that the uncertainty due to the radiative corrections of the second and third group is very small. So, the main source of uncertainty can be reduced to the experimental uncertainty in the electronic decay widths of charmonium.

Within light cone formalism in addition to QCD radiative corrections there are power corrections to the leading order approximation of $1/s$ expansion. These corrections appear due to the contribution of the fragmentation and non-fragmentation diagrams. In Table II the error due to this corrections are estimated as $M^2/E^2\sim40\%$. To reduce this uncertainty let us consider $O(1/s^2)$ corrections to the leading order result.

III. NEXT-TO-LEADING ORDER CONTRIBUTION IN $1/s$ EXPANSION.

To calculate the cross section at $O(1/s^2)$ approximation of light cone formalism let us look to Table I. It is seen from this table that there are two contributions at this level of accuracy. The first one is due to the fragmentation diagrams with the following polarizations of the mesons $\lambda_1 = \pm 1,\ \lambda_2 = 0$ and $\lambda_1 = 0,\ \lambda_2 = \pm 1$. It causes no difficulties to calculate these cross sections

$$\frac{d\sigma^{fr}_{\pm 1,0}}{dx} = \frac{16\pi^3\alpha^4 q_c^4 f_1^2 f_2^2\sqrt{\lambda}}{sM_1^4M_2^4}\frac{2r_2}{((1-x^2)\lambda+4r_1r_2)^2}\left((r_1-r_2+1)^2x^4+(6r_1^2-2r_2^2+4r_2-2)x^2+(r_1+r_2-1)^2\right),$$
$$\frac{d\sigma^{fr}_{0,\pm 1}}{dx} = \frac{16\pi^3\alpha^4 q_c^4 f_1^2 f_2^2\sqrt{\lambda}}{sM_1^4M_2^4}\frac{2r_1}{((1-x^2)\lambda+4r_1r_2)^2}\left((r_2-r_1+1)^2x^4+(6r_2^2-2r_1^2+4r_1-2)x^2+(r_1+r_2-1)^2\right). \quad (7)$$

The second contribution arises from the interference between the fragmentation and non-fragmentation diagrams with polarization of vector mesons $\lambda_1 = \pm 1,\ \lambda_2 = \mp 1$. It is not difficult to find the amplitude of the non-fragmentation diagrams (Fig. 1c,d) for these polarization of vector mesons using LCF

$$M = -\frac{2^9\pi^2\alpha^2 q_c^2 g_1(\mu)g_2(\mu)}{3s^3}\big[(2(p_2k_1)-2(p_1k_1))(\epsilon_1^*\epsilon_2^*)\bar u(k_2)\hat p_1 u(k_1)+$$
$$+s(\epsilon_1^*k_1)\bar u(k_2)\hat\epsilon_2^* u(k_1)+s(\epsilon_2^*k_1)\bar u(k_2)\hat\epsilon_1^* u(k_1))\big]A(\cos\theta,\mu), \quad (8)$$

where $\bar u(k_2), u(k_1)$ are positron and electron bispinors, $\epsilon_1^*, \epsilon_2^*$ are polarization vectors of charmonia. The constants $g_i(\mu)$ are defined as follows:

$$\langle V_i(p_i,\lambda_i)|\bar C\sigma_{\mu\nu}C|0\rangle_\mu = g_i(\mu)\big(\epsilon^*_\mu p_\nu - \epsilon^*_\nu p_\mu\big). \quad (9)$$

It should be noted that the operator $\bar C\sigma_{\alpha\beta}C$ is not renormalization group invariant. For this reason the constant g_i depends on scale as

$$g_i(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{4}{3b_0}} g_i(\mu_0). \quad (10)$$

The function $A(x,\mu)$ is defined as

$$A(x,\mu) = \frac{1}{8}\int d\xi_1 d\xi_2 \frac{\phi_1(\xi_1,\mu)\phi_2(\xi_2,\mu)}{(1+\xi_1\xi_2)^2-(\xi_1+\xi_2)^2x^2}\left[\frac{1}{x_1y_1}+\frac{1}{x_2y_2}\right], \quad (11)$$

here x_1, x_2 are the fractions of momentum carried by quark and antiquark in the first meson, y_1, y_2 are the fractions of momentum carried by quark and antiquark in the second meson, $\xi_1 = x_1 - x_2, \xi_2 = y_1 - y_2$, $\phi_1(\xi_1,\mu), \phi_2(\xi_2,\mu)$ are leading twist light cone distribution amplitudes of vector charmonium mesons with transverse polarization.

Now some comments on formula (8) are in order:

1. Formula (8) is the leading twist contribution to the amplitude of the diagrams shown in Fig 1c,d. For this reason it contains only the distribution amplitudes $\phi_1(\xi_1), \phi_2(\xi_2)$ of the leading twist.

2. It is seen from (8) that the amplitude of the non-fragmentation diagrams depends on the distribution amplitudes $\phi_i(\xi_i,\mu)$ of vector mesons. If infinitely narrow distribution amplitudes $\phi_i(\xi_i,\mu) = \delta(\xi_i)$ are substituted to formula (8), than NRQCD result for the amplitude will be reproduced. If real distribution amplitudes $\phi_i(\xi_i,\mu)$ are taken at scale $\mu \sim m_c$, than formula (8) will resum the relativistic corrections to the cross section up to $O(1/s^3)$ terms. To resum the relativistic and leading logarithmic corrections simultaneously one must take the distribution amplitudes $\phi_i(\xi_i,\mu)$ and the constants $g_i(\mu)$ at the characteristic scale of the process $\mu \sim \sqrt{s}$. The calculation of the cross sections will be done at scale $\mu = E = \sqrt{s}/2$.

The calculation of $\sigma^{int}_{\pm 1,\mp 1}$ will be done as follows. For the non-fragmentation diagrams the amplitude will be taken in form (8). For the fragmentation diagrams the exact expression for the amplitudes will be taken (see discussion in the previous section). Then the standard procedure for the calculation of the $\sigma^{int}_{\pm 1,\mp 1}$ will be applied. Thus one gets the result

$$\frac{d\sigma^{int}_{\pm 1,\mp 1}}{dx} = -\frac{2^8\pi^3\alpha^4 q_c^4 f_1 f_2 g_1(E)g_2(E)\sqrt{\lambda}}{3s^2M_1^2M_2^2}\left(\frac{1-r_1-r_2}{(1-x^2)\lambda+4r_1r_2}\right)(1-x^4)A(x,E), \quad (12)$$

The total cross section has the form

$$\frac{d\sigma}{dx} = 2\frac{d\sigma^{fr}_{\pm 1,\mp 1}}{dx}+2\frac{d\sigma^{fr}_{\pm 1,0}}{dx}+2\frac{d\sigma^{fr}_{0,\pm 1}}{dx}+2\frac{d\sigma^{int}_{\pm 1,\mp 1}}{dx}+O\left(\frac{1}{s^3}\right) \quad (13)$$

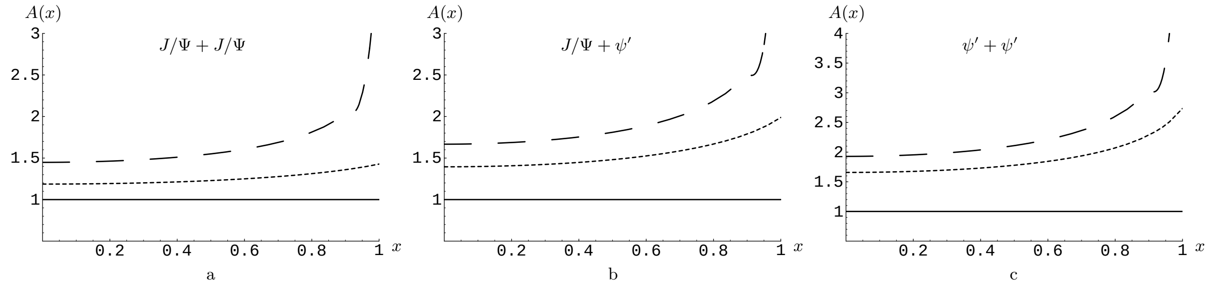

FIG. 2: The plots of the functions $A(x,\mu)$ for the processes $e^+e^- \to J/\Psi + J/\Psi$ (**fig. a**), $J/\Psi + \psi'$ (**fig. b**), $\psi' + \psi'$ (**fig. c**). Solid lines correspond to the leading order NRQCD predictions for the functions $A(x,\mu)$. Small dashed lines represent the functions $A(x,\mu)$ if the relativistic corrections are taken into the account. Long dashed lines represent the functions $A(x,\mu)$ if the relativistic corrections and leading logarithmic radiative corrections are taken into the account simultaneously.

IV. NUMERICAL RESULTS AND DISCUSSION.

From formulas (12), (13) one sees that the cross section depends on the function $A(x,\mu)$. This function takes into account internal motion of quark-antiquark pairs in mesons inside the hard part of the amplitude. In addition, this function resums the leading logarithmic radiative corrections to the amplitude. If one ignores both of these effects the function $A(x,\mu)$ equals unity, what corresponds to the leading order approximation of NRQCD. It is interesting to study how the relativistic and leading logarithmic radiative corrections can change the leading order NRQCD predictions. To do this one needs to know the distribution amplitudes $\phi_i(x,\mu)$ of $1S$ and $2S$ states vector charmonium mesons. These distribution amplitudes were studied in papers [18, 19, 20]. The calculation of the functions $A(x,\mu)$ and the cross sections of the processes considered will be done using the models of distribution amplitudes proposed in these papers:

$$\begin{aligned} \phi_{1S}(\xi,\mu\sim M_c) &\sim (1-\xi^2)\ \mathrm{Exp}\left[-\frac{\beta}{1-\xi^2}\right] \\ \phi_{2S}(\xi,\mu\sim M_c) &\sim (1-\xi^2)(\alpha+\xi^2)\ \mathrm{Exp}\left[-\frac{\beta}{1-\xi^2}\right], \end{aligned} \tag{14}$$

where $M_c = 1.2$ GeV is the QCD sum rules mass parameter. For $1S$ charmonium state the constant β can vary within the interval 3.8 ± 0.7. For $2S$ charmonium state the constants α and β can vary within the intervals $0.03^{+0.32}_{-0.03}$ and $2.5^{+3.2}_{-0.8}$ correspondingly.

Having models of distribution amplitudes (14), it causes no difficulties to calculate the functions $A(x,\mu)$. The plots of the functions $A(x,\mu)$ for the processes $e^+e^- \to J/\Psi + J/\Psi, J/\Psi + \psi', \psi' + \psi'$ are shown in Fig. 2a, b, c. Solid lines correspond to the leading order NRQCD predictions for the functions $A(x,\mu)$. Small dashed lines represent the functions $A(x,\mu)$ if relativistic corrections are taken into the account. Long dashed lines represent the functions $A(x,\mu)$ if the relativistic corrections and leading logarithmic radiative corrections are are taken into the account simultaneously.

From Fig. 2 one sees that the relativistic and leading logarithmic radiative corrections not only can change characteristic value of the function $A(x,\mu)$ but they can also considerably modify the shape of this function. This statement is especially true for the production of higher charmonia such as ψ' meson, since the relativistic corrections play very important role in this case.

Now let us calculate the cross sections of the processes under consideration. To do this one needs the values of the constants $g_i(\mu)$ at some scale. Unfortunately, it is rather difficult to determine these constants directly from the experiment. The values of these constants can be obtained in the framework of NRQCD(see Appendix):

$$g_1^2(M_{J/\Psi}) = 0.144 \pm 0.016\ \ \mathrm{GeV}^2, \qquad g_2^2(M_{J/\Psi}) = 0.068 \pm 0.022\ \ \mathrm{GeV}^2. \tag{15}$$

The plots of the differential cross sections $d\sigma/dx$ for the processes $e^+e^- \to J/\Psi + J/\Psi, J/\Psi + \psi', \psi' + \psi'$ are shown in Fig. 3a, b, c. The values of the cross sections for the processes under study are shown in Table II.

There are different sources of uncertainty to the results obtained in this paper. The uncertainties in the fragmentation contribution were discussed above. The uncertainties in the non-fragmentation contribution can be divided into the following groups:

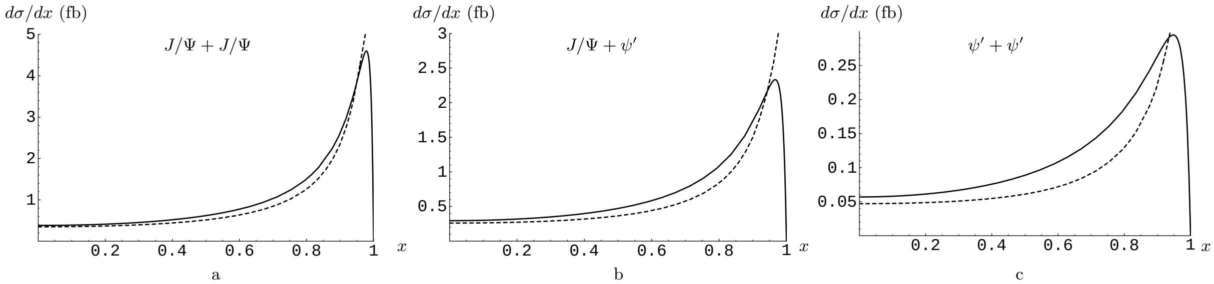

FIG. 3: The plots of the differential cross sections $d\sigma/dx$ ($x=\cos\theta$) for the processes $e^+e^-\to J/\Psi+J/\Psi$ (**fig. a**), $J/\Psi+\psi'$ (**fig. b**), $\psi'+\psi'$ (**fig. c**). Solid lines correspond to the $O(1/s)$ contributions to the cross sections. Small dashed lines represent the cross sections at $O(1/s^2)$ approximation.

1. The uncertainty in the models of the distribution amplitudes $\phi_i(x,\mu)$, which can be estimated through the variation of the parameters of these models (14). Thus it is not difficult to show that the error in the cross sections due to the uncertainty in the model of the distribution amplitude of J/Ψ meson is not very important (about few percents) and it will be ignored further. The error due the uncertainty in the model of the distribution amplitude of ψ' meson is about 10% of the interference contribution for the process $e^+e^-\to J/\Psi+\psi'$ and about 20% of the interference contribution for the process $e^+e^-\to\psi'+\psi'$.

2. The uncertainty due to the radiative corrections to the non-fragmentation diagrams. In the approach applied in this paper the leading logarithmic radiative corrections to the amplitude have been resummed in the distribution amplitudes. This fact allows us to estimate the rest of the radiative corrections as $\sim\alpha_s(E)\sim 20\%$. It should be noted that if one does not resum the leading logarithmic radiative corrections the error of the calculation must be estimated as $\sim\alpha_s(E)\log s/M^2_{J/\Psi}\sim 50\%$ instead of $\sim\alpha_s(E)\sim 20\%$ as it was done in paper [28].

3. The uncertainty due to the power corrections. This uncertainty is determined by the $O(1/s^3)$ terms. One can estimate this source of uncertainty as $\sim M^2/E^2\sim 40\%$.

4. The uncertainty in the values of constants (15).

Adding all these uncertainties in quadrature one gets the total error of the calculations.

Now it is interesting to compare the results for the cross sections with experimental data. The cross sections of the processes considered in this paper were measured at Belle collaboration [2]. Unfortunately, only the upper bound on these cross sections were determined:

$$\begin{aligned}
&\sigma(e^+e^-\to J/\Psi J/\Psi)\times Br_{>2}(J/\Psi)<9.1\text{ fb} && 90\text{ \% CL},\\
&\sigma(e^+e^-\to J/\Psi\psi')\times Br_{>2}(\psi')<13.3\text{ fb} && 90\text{ \% CL},\\
&\sigma(e^+e^-\to J/\Psi\psi')\times Br_{>0}(J/\Psi)<16.9\text{ fb} && 90\text{ \% CL},\\
&\sigma(e^+e^-\to \psi'\psi')\times Br_{>0}(\psi')<5.2\text{ fb} && 90\text{ \% CL},
\end{aligned}\tag{16}$$

where $Br_{>2}(V)$ denotes the branching fraction of V into final states with more than two charged tracks, $Br_{>0}(V)$ is the branching fraction of V into final states containing charged tracks. Unfortunately, the values of the $Br_{>0,2}(V)$ are unknown. However, one can expect that the values of the $Br_{>0}(J/\Psi), Br_{>2}(J/\Psi), Br_{>0}(\psi')$ are rather close to unity, what allows us to estimate $\sigma(e^+e^-\to J/\Psi J/\Psi)<9.1$ fb, $\sigma(e^+e^-\to J/\Psi\psi')<16.9$ fb and $\sigma(e^+e^-\to\psi'\psi')\times Br_{>0}(\psi')<5.2$ fb. These estimations are in agreement with the values of the cross sections obtained in this paper.

From the results shown in Fig. 3 and in Table II one sees that $O(1/s^2)$ contribution does not change greatly LO results for the process $e^+e^-\to J/\Psi J/\Psi$. The smallness of $O(1/s^2)$ contribution for this process can be explained as follows. At $O(1/s^2)$ approximation there are two contributions to the cross sections: the fragmentation diagrams and the interference of the fragmentation and non-fragmentation diagrams. These contributions are very near to each other and have different signs. So, due to partial cancellation $O(1/s^2)$ contribution to the cross section is suppressed. If one further considers the production of higher charmonia, the value of the wave functions at the origin for these states are smaller and the total cross section becomes smaller. However, due to the relativistic and radiative corrections, collected in the functions A, the contribution of the non-fragmentation diagrams is enhanced (see Fig. 2). For this reason $O(1/s^2)$ contribution plays more significant role for the processes $e^+e^-\to J/\Psi\psi'$ and it is very important for the process $e^+e^-\to\psi'\psi'$.

It should be noted here that light cone formalism can be applied to study the production of light meson, for instance, ρ mesons. In this case, all formulas derived in this paper remain valid. The $O(1/s^2)$ contribution can be estimated as

V_1 V_2	σ^{LO}(fb)	$\sigma^{LO}_{\lvert cos\theta\rvert<0.8}$(fb)	σ^{NLO}(fb)	$\sigma^{NLO}_{\lvert cos\theta\rvert<0.8}$(fb)	$\sigma^{[28]}$(fb)	$\sigma^{[28]}_{\lvert cos\theta\rvert<0.8}$(fb)	$\sigma^{[29]}$(fb)
J/Ψ J/Ψ	2.12 ± 0.85	1.02 ± 0.41	2.02 ± 0.25	0.86 ± 0.17	1.69 ± 0.35	0.60 ± 0.24	$1.8 - 2.3$
J/Ψ ψ'	1.43 ± 0.57	0.77 ± 0.31	1.32 ± 0.16	0.61 ± 0.16	0.95 ± 0.36	0.33 ± 0.24	–
ψ' ψ'	0.24 ± 0.10	0.14 ± 0.06	0.20 ± 0.06	0.10 ± 0.05	0.11 ± 0.09	0.04 ± 0.06	–

TABLE II: The cross sections of the processes $e^+e^- \to J/\Psi J/\Psi,\ J/\Psi\psi',\ \psi'\psi'$. The second column contains the cross sections at the leading order approximation of $1/s$ expansion. The third column contains the differential cross sections integrated over the region $|\cos\theta| < 0.8$. The values of the cross sections at $O(1/s^2)$ approximation are shown in the forth and fifth columns. The sixth and seventh columns contain the results obtained in paper [28]. The results obtained in paper [29] are shown in the last column.

$\sim M^2/s$, which is very small value for light mesons. For this reason, one can state that the values of the cross sections of double light meson production obtained in the approximation when only fragmentation diagrams are taken into account [27, 28] are rather reliable.

At the end of this section it is interesting to compare the results obtained in this paper with the results obtained in other papers devoted to the calculation of the same processes. It has already been noted that the processes of double vector mesons production were considered in the following papers [24, 25, 26, 27, 28, 29]. In papers [26, 27] only the contribution arising from the fragmentation diagrams was considered. The results of their calculation are in good agreement with results obtained at LO approximation. The papers [24, 25, 28], were written by the same group of authors, so it is reasonable to consider the results obtained in the last one [28]. In this paper the calculation was done at $O(v^2)$ approximation of NRQCD. The results obtained in paper [28] are shown in the last two columns of Table II. It is seen that within the error of the calculation this results are in agreement with that obtained in this paper.

Now let us consider the results obtained in paper [29]. In this paper the radiative corrections to the process $e^+e^- \to J/\Psi J/\Psi$ were calculated. The result of the calculation can be written in the following form

$$\sigma^1 = \sigma^0(1 + \frac{\alpha_s}{\pi}K), \tag{17}$$

where σ^1 is the cross section with the account of radiative corrections, σ^0 is the cross section without radiative corrections, the factor $K = -11.19$ for the pole mass of c-quark equals to 1.5 GeV. From this one sees that the radiative corrections to the cross section are very large, what leads to sizable reduction of the cross section σ^0. As concerns the results obtained in this paper, it is seen from Tab. II and from Fig. 2, 3 that the leading logarithmic radiative corrections do not change the cross section greatly. To the first sight, one can think that this contradicts to results [29]. However, there is no contradiction between these results. To see this let us consider the results of paper [29] in more detail. One of the input parameter for the calculation of the σ^0 is the wave function at the origin $|R_s^{J/\Psi}(0)|^2$. In paper [29] this parameter was determined from the electron decay width of J/Ψ through the following formula

$$\Gamma_{ee} = \left(1 - \frac{16}{3}\frac{\alpha_s}{\pi}\right)\frac{4\alpha^2 e_c^2}{M^2_{J/\Psi}}|R_s^{J/\Psi}(0)|^2. \tag{18}$$

It seen that this formula determines the value of $|R_s^{J/\Psi}(0)|^2$ taking into the account α_s correction. As the result some part of the radiative corrections is present in σ^0, which according to definition is the leading order in α_s quantity. So, the authors of this paper separated the whole radiative corrections into two parts which nearly coincide but have different sign. If one now merges these two parts, the following result can be obtained

$$\sigma^1 = \sigma^0(1 + \frac{32}{3}\frac{\alpha_s}{\pi} + (-11.19)\frac{\alpha_s}{\pi}) = \sigma^0(1 + (-0.52)\frac{\alpha_s}{\pi}), \tag{19}$$

what is in agreement with the results obtained in this paper. The value of the cross section of the process $e^+e^- \to J/\Psi J/\Psi$ obtained in paper [29] is presented in Table II. The variation of the cross section is due to the variation of the pole mass of c quark $1.4 - 1.5$ GeV.

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APPENDIX A: THE CALCULATION OF THE CONSTANTS $g_i(\mu)$.

To calculate the values of the constants $g_i(\mu)$ (9) one can apply NRQCD formalism. At $O(v^2)$ approximation of NRQCD the constants $g_i(\mu)$ and f_i can be written as follows [10, 31]

$$\begin{aligned} f_i^2 &= \langle V_i(\epsilon)|\chi^+(\vec\sigma\vec\epsilon)\varphi|0\rangle\langle 0|\chi^+(\vec\sigma\vec\epsilon)\varphi|V_i(\epsilon)\rangle \times \left(1-\frac{16}{3}\frac{\alpha_s}{\pi}-\frac{1}{3}\langle v^2\rangle_i\right), \\ g_i^2(\mu) &= \langle V_i(\epsilon)|\chi^+(\vec\sigma\vec\epsilon)\varphi|0\rangle\langle 0|\chi^+(\vec\sigma\vec\epsilon)\varphi|V_i(\epsilon)\rangle \times \left(1-\frac{16}{3}\frac{\alpha_s}{\pi}-\frac{2}{3}\frac{\alpha_s}{\pi}\log\frac{\mu^2}{m_c^2}-\frac{2}{3}\langle v^2\rangle_i\right), \end{aligned} \tag{A1}$$

where

$$\langle v^2\rangle_i = -\frac{1}{m_c^2}\frac{\langle 0|\chi^+(\vec\sigma\vec\epsilon)(\overleftrightarrow{\mathbf{D}})^2\varphi|V_i(\epsilon)\rangle}{\langle 0|\chi^+(\vec\sigma\vec\epsilon)\varphi|V_i(\epsilon)\rangle}. \tag{A2}$$

The calculation of the constants $g_i(\mu)$ will be done at scale $\mu = M_{J/\Psi}$. To diminish the error of the calculation let us consider the ratio $g_i^2(M_{J/\Psi})/f_i^2$. At the same level of accuracy it can be written as follows

$$\frac{g_i^2(M_{J/\Psi})}{f_i^2} = \left(1-\frac{2}{3}\frac{\alpha_s}{\pi}\log\frac{M_{J/\Psi}^2}{m_c^2}-\frac{\langle v^2\rangle_i}{3}\right). \tag{A3}$$

The values of the constants $g_i(M_{J/\Psi})$ will be calculated with the following set of parameters: $\alpha_s(M_{J/\Psi}) = 0.25$, $\langle v^2\rangle_{J/\Psi} = 0.25$ [32], $\langle v^2\rangle_{\psi'} = 0.54$ [20]. To estimate the error of the calculation one should take into account that within NRQCD the constant is double series in relativistic and radiative corrections. At NNLO approximation one has relativistic corrections $\sim \langle v^2\rangle^2$, radiative corrections to the short distance coefficient of the operator $\langle 0|\chi^+(\vec\sigma\vec\epsilon)\varphi|V_i(\epsilon)\rangle$ $\sim \alpha_s^2$ and radiative corrections to the short distance coefficient of the operator $\langle 0|\chi^+(\vec\sigma\vec\epsilon)(\overleftrightarrow{\mathbf{D}})^2\varphi|V_i(\epsilon)\rangle$ that can be estimated as $\sim \alpha_s\langle v^2\rangle$. Adding all these uncertainties in quadrature one can estimate the error of the calculation. Thus one gets

$$\begin{aligned} g_1^2(M_{J/\Psi}) &= 0.144 \pm 0.016 \ \ \mathrm{GeV}^2, \\ g_2^2(M_{J/\Psi}) &= 0.068 \pm 0.022 \ \ \mathrm{GeV}^2. \end{aligned} \tag{A4}$$

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